MANGALORE UNIVERSITY
DEPARTMENT OF MATHEMATICS

M.Sc. Mathematics Choice Based Credit System (Semester Scheme) Programme from the academic year 2016-17

Preamble

The syllabi for the M.Sc. Mathematics Programme in use at present were introduced from the academic year 2011-2012. As per the directions and guidelines of The University Grants Commission, and also with instructions from The Higher Education Council of Government of Karnataka, The Mangalore University has recently framed the regulations governing the Choice Based Credit System for the two years (four semesters) post graduate degree programmes **(called CBCS-PG)** so as to enable its programmes to be on par with global standards. Hence the following revised and restructured syllabi for the M.Sc. Mathematics Programme have been prepared as per the new regulations of the University. In the syllabi, the first paper in each of the second and the third semesters is an "Open Elective" paper, which is offered only to the students of other departments. Also, to have better flexibility in introducing the courses, additional optional courses are offered under "Soft Core" category. The syllabi take into consideration the recommendations of U.G.C. Curriculum Development Committee and it is meant to be introduced from the academic year 2016-2017.

Programme Outcome:

- Provide a strong foundation in different areas of Mathematics, so that the students can compete with their contemporaries and excel in the various careers in Mathematics.
- Develop abstract mathematical thinking.
- Motivate and prepare the students to pursue higher studies and research, thus contributing to the ever increasing academic demands of the country.
- Enrich the students with strong communication and interpersonal skills, broad knowledge and an understanding of multicultural and global perspectives, to work effectively in multidisciplinary teams, both as leaders and team members.
- Facilitate integral development of the personality of the student to deal with ethical and professional issues, and also to develop ability for independent and lifelong learning.

Programme Specific Outcome:

- Students will demonstrate in-depth knowledge of Mathematics, both in theory and application. They develop problem-solving skills and apply them independently to problems in pure and applied mathematics.
- Students will attain the ability to identify, formulate and solve challenging problems in Mathematics. They assimilate complex mathematical ideas and arguments.
- Students will be able to analyse complex problems in Mathematics and propose solutions using research based knowledge
- Students will be able to work individually or as a team member or leader in uniform and multidisciplinary settings.

- Students will develop confidence for self-education and ability for lifelong learning. Adjust themselves completely to the demands of the growing field of Mathematics by lifelong learning.
- Effectively communicate about their field of expertise on their activities, with their peer and society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations.
- Crack lectureship and fellowship exams approved by UGC like CSIR – NET and SET.

Consolidated List of Courses offered:

Hard Core Courses

First Semester:

1. MTH 401 Algebra – I
2. MTH 402 Linear Algebra – I
3. MTH 403 Real Analysis – I

Second Semester:

4. MTH 452 Algebra – II
5. MTH 453 Real Analysis – II
6. MTH 454 Topology

Third Semester:

7. MTH 502 Complex Analysis – I
8. MTH 503 Measure and Integration
9. MTH 504 Multivariate Calculus and Geometry

Fourth Semester:

10. MTP 551 Project Work
11. MTH 552 Complex Analysis – II
12. MTH 553 Functional Analysis

Soft Core Courses

1. MTS 404 Numerical Analysis
2. MTS 405 Number Theory
3. MTS 455 Linear Algebra – II
4. MTS 456 Ordinary Differential Equations
5. MTS 505 Advanced Numerical Analysis
6. MTS 506 Commutative Algebra
7. MTS 507 Graph Theory
8. MTS 508 Lattice Theory
9. MTS 509 Fluid Mechanics
10. MTS 510 Theory of Partitions
11. MTS 554 Partial Differential Equations
12. MTS 555 Advanced Topology
13. MTS 556 Advanced Discrete Mathematics
14. MTS 557 Algebraic Number Theory
15. MTS 558 Calculus of Variations and Integral Equations
16. MTS 559 Mathematical Statistics

Open Elective Courses

1. MTE 451 Discrete Mathematics and Applications
2. MTE 501 Differential Equations and Applications

A. The following shall be the Courses of study in the four semesters M.Sc. Mathematics Programme (CBCS-PG) from the academic year 2016-2017.

I Semester

Course Code	Course	Hard Core/ Soft Core/ Open Elective	Credits
MTH 401	Algebra – I	HC	5
MTH 402	Linear Algebra – I	HC	5
MTH 403	Real Analysis – I	HC	5
MTS 404	Numerical Analysis	SC	4
MTS 405	Number Theory	SC	4

II Semester

In this semester, the course 'MTE 451' is an "Open Elective Course" which is offered only to students of other departments. The other five courses are offered to the students of the department.

Course Code	Course	Hardcore/ Soft Core/ Open Elective	Credits
MTE 451	Discrete Mathematics and Applications	OE	3
MTH 452	Algebra – II	HC	4
MTH 453	Real Analysis – II	HC	4
MTH 454	Topology	HC	5
MTS 455	Linear Algebra – II	SC	4
MTS 456	Ordinary Differential Equations	SC	4

III Semester

In this semester, the course 'MTE 501' is an "Open Elective Course" which is offered only to students of other departments. The other courses are offered to the students of the department. The hard core courses MTH 502, MTH 503 and MTH 504 are compulsory and the student can choose any two soft core courses from MTS 505 to MTS 510. Also a project work, which is compulsory for every student, involves self study to be carried out by the student (on a research problem of current interest or on an advanced topic not covered in the syllabus) under the guidance of a faculty member. Project work shall be initiated in the third semester itself and the project report (dissertation) shall be submitted at the end of the fourth semester.

Course Code	Course	Hard Core/ Soft Core/ Open Elective	Credits
MTE 501	Differential Equations and Applications	OE	3
MTH 502	Complex Analysis – I	HC	5
MTH 503	Measure and Integration	HC	5
MTH 504	Multivariate Calculus and Geometry	HC	4
MTS 505	Advanced Numerical Analysis	SC	4
MTS 506	Commutative Algebra	SC	4
MTS 507	Graph Theory	SC	4
MTS 508	Lattice Theory	SC	4
MTS 509	Fluid Mechanics	SC	4
MTS 510	Theory of Partitions	SC	4

IV Semester

In this semester, the course MTP 551 is a project work which the student has taken up under the guidance of a faculty member in the third semester itself. Each student has to submit a project report (dissertation) at the end of the fourth semester. The hard core courses MTH 552 and MTH 553 are compulsory and the student can choose any two soft core courses from MTS 554 to MTS 559.

Course Code	Course	Hard Core/ Soft Core/ Open Elective	Credits
MTP 551	Project Work	Project	4
MTH 552	Complex Analysis – II	HC	4
MTH 553	Functional Analysis	HC	4
MTS 554	Partial Differential Equations	SC	4
MTS 555	Advanced Topology	SC	4
MTS 556	Advanced Discrete Mathematics	SC	4
MTS 557	Algebraic Number Theory	SC	4
MTS 558	Calculus of Variations and Integral Equations	SC	4
MTS 559	Mathematical Statistics	SC	4

B. Scheme of Instruction and Examination

I Semester

Course Code	Instruction hours per week	Credits	Duration of Exam. in hours	University Exam. Max.Marks	Internal Assessment Max.Marks	Total Marks
MTH 401	5	5	3	70	30	100
MTH 402	5	5	3	70	30	100
MTH 403	5	5	3	70	30	100
MTS 404	4	4	3	70	30	100
MTS 405	4	4	3	70	30	100

II Semester

Course Code	Instruction hours per week	Credits	Duration of Exam. in hours	University Exam. Max.Marks	Internal Assessment Max.Marks	Total Marks
MTE 451	3	3	3	70	30	100
MTH 452	4	4	3	70	30	100
MTH 453	4	4	3	70	30	100
MTH 454	5	5	3	70	30	100
MTS 455	4	4	3	70	30	100
MTS 456	4	4	3	70	30	100

III Semester

Course Code	Instruction hours per week	Credits	Duration of Exam. in hours	University Exam. Max. Marks	Internal Assessment Max. Marks	Total Marks
MTE 501	3	3	3	70	30	100
MTH 502	5	5	3	70	30	100
MTH 503	5	5	3	70	30	100
MTH 504	4	4	3	70	30	100
MTS 505	4	4	3	70	30	100
MTS 506	4	4	3	70	30	100
MTS 507	4	4	3	70	30	100
MTS 508	4	4	3	70	30	100
MTS 509	4	4	3	70	30	100
MTS 510	4	4	3	70	30	100

IV Semester

Course Code	Instruction hours per week	Credits	Duration of Exam. in hours	University Exam. Max.Marks	Internal Assessment Max.Marks	Total Marks
MTP 551	4	4	-	70	30	100
MTH 552	4	4	3	70	30	100
MTH 553	4	4	3	70	30	100
MTS 554	4	4	3	70	30	100
MTS 555	4	4	3	70	30	100
MTS 556	4	4	3	70	30	100
MTS 557	4	4	3	70	30	100
MTS 558	4	4	3	70	30	100
MTS 559	4	4	3	70	30	100

Tutorials: There shall be at least 3 hours of tutorials per week for each course having 5 credits; and the minimum number of hours of tutorials per week for courses with less than 5 credits shall be considered proportionately.

Scheme of Evaluation for Internal Assessment Marks

1. **Theory Course**:
   Each Theory Course shall carry 30 marks for internal assessment based on two tests of 90 minutes duration each.

2. **Project Work:**
   Project Work shall carry 30 marks for internal assessment based on two presentations by the candidate before a panel of faculty members of the department.

Pattern of Semester Examination

1. **Theory Paper**:
   Each question paper for the theory course shall contain **EIGHT** questions out of which **FIVE** are to be answered. All questions carry equal marks.

2. **Project Report:**
   The evaluation of a project report is by two examiners as per the regulations.

C. Syllabi of Each Semester

I SEMESTER

MTH 401	Algebra – I	5 Credits (60 hours)

Course Outcome: To introduce the concepts and to develop working knowledge on fundamentals of algebra. Students will have the knowledge and skills to apply the concepts of the course in pattern recognition in the field of computer science and also for diverse situations in physics, chemistry and other streams. This course is a foundation for next course in Algebra.

Course Specific Outcome: At the end of the course students will have the knowledge and skills to understand, explain in depth and apply the fundamental concepts-

- Groups
- Structure of Groups
- Rigid motions, isometries
- Rings and integral domains.

Unit I - Groups and Subgroups: (Ref. [1])

Binary operations, Isomorphic binary operations, Groups, Subgroups, Cyclic groups, Generating sets and Cayley digraphs, Groups of permutations, Orbits, Cycles and alternating groups, Cosets and Lagrange's theorem.

15 Hours

Unit II - Product Groups, Homomorphism and Quotient Groups: (Ref. [1])

Direct products and finitely generated abelian groups, Homomorphisms, Factor groups, Factor group computations and simple groups, Isomorphism theorems, Series of groups.

15 Hours

Unit III - Advanced Group Theory: (Ref. [2])

Symmetry of plane figures, Isometries, Isometries of the plane, Finite groups of orthogonal operators on the plane.
Group actions on a set, Applications of group actions to counting, Cayley's theorem, The class equation, p-Groups, Conjugation in the symmetric group, Normalizers, The Sylow theorems, The groups of order 12.

22 Hours

Unit IV - Rings and Fields: (Ref. [1])

Definitions of rings, subrings, integral domains, fields and their basic properties, Homomorphisms and Factor Rings, Prime and Maximal Ideals. Fields of quotients of an integral domain, Rings of Polynomials.

8 Hours

References:

1. J. B. Fraleigh – A First Course in Abstract Algebra, Addison Wesley, 7th Edition, 2003.
2. Michael Artin – Algebra, Prentice Hall of India 2nd Edition, 2013.
3. I. N. Herstein – Topics in Algebra, John Wiley & Sons, 2nd Edition, 2006.
4. Joseph A. Gallian – Contemporary Abstract Algebra, Cengage Learning India, 8th Edition, 2013.
5. Paul B. Garrett – Abstract Algebra, CRC press, 2007.
6. Thomas W. Hungerford – Algebra, Springer, 2004.
7. David S. Dummit and Richard M. Foote – Abstract Algebra, Wiley, 3rd Edition, 2004.
8. Serge Lang – Algebra, Springer, 3rd Edition, 2005.

MTH 402	Linear Algebra – I	5 Credits (60 hours)

Course Outcome: Students will have the knowledge and skills to explain the fundamental concepts of Matrix Operations, vector spaces, Linear Operators, Eigenvectors, The characteristic polynomial, Jordan form, the concepts Orthogonal matrices and Rotations, The matrix exponential, which is use to solve differential equations arsing in the fields like physics, chemistry, economics and also in biology. This course is a foundation for next course in Linear algebra.

Course Specific Outcome: At the end of the course students will have the knowledge and skills

- To develop techniques to work with matrices, Jordan form etc.
- To enhance one's skills in applying matrices to solve differential equations.
- To acquaint knowledge in the theory of vector spaces
- To acquaint knowledge in the theory of linear transformations.

Unit I - Matrix Operations:

Recapitulation of the basic operations, Block multiplication, Matrix units, Row reduction, The matrix transpose, Permutation matrices, Determinants, Other formulas for Determinant, The Cofactor matrix. 14 Hours

Unit II - Vector Spaces:

Subspaces of R^n, Fields, Vector Spaces, Bases and dimension. Computing with bases, Direct sums, Infinite Dimensional spaces. 18 Hours

Unit III - Linear Operators:

The dimension formula, The matrix of a linear transformation, Linear Operators, Eigenvectors, The characteristic polynomial, Triangular and Diagonal forms, Jordan form.

20 Hours

Unit IV – Applications of Linear Operators:

Orthogonal matrices and Rotations, Cayley-Hamilton Theorem, The matrix exponential,

8 Hours

References:

1. Michael Artin – Algebra, Prentice Hall of India, 2rd Edition, 2013.
2. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence – Linear Algebra, Prentice Hall of India, 4th Edition, 2014.
3. K. Hoffmann and R. Kunz – Linear Algebra, Prentice Hall of India, 2nd Edition, 2013.
4. S. Lang – Linear Algebra, Addison Wesley, London, 1970.
5. Larry Smith – Linear Algebra, Springer Verlag, 3rd Edition, 1998.
6. Gilbert Strang – Linear Algebra and its Applications, Cengage Learning, 4th Edition, 2006.
7. S. Kumaresan – Linear Algebra - A Geometric Approach, PHI, 2003.

MTH 403	Real Analysis – I	5 Credits (60 hours)

Course Outcome: Students will have the knowledge and skills to explain the fundamental concepts of the real number system, Perfect sets, Connected sets, explain the concepts of convergent sequences, subsequences, Cauchy sequences, Series, the derivative of a real function, Mean value theorems, L'Hospital's rule, Taylor's theorem and its applications, differential equations and more generally in mathematical analysis.

Course Specific Outcome: At the end of the course students will have the knowledge and skills

- To study the real number system and their properties in detail.
- To develop skills to work with sequences in arbitrary metric spaces.
- To develop skills to work with series of real numbers.
- To study the concepts of continuous functions and differentiable functions.

Unit I

The real and complex number system: Introduction, Ordered sets, Fields, The real field, The extended real number system, The complex field, Euclidean spaces, Inequalities.

Basic topology: Finite, countable and uncountable sets, Metric spaces, Compact sets, Perfect sets, Connected sets. 20 Hours

Unit II - Numerical sequences and Series:

Convergent sequences, Subsequences, Cauchy sequences, Upper and lower limits, Some special sequences, Series, Series of non-negative terms, The number e, The root and ratio tests, Power series, Summation by parts, Absolute convergence, Addition and multiplication of series, Rearrangements. 15 hours

Unit III - Continuity:

Limits of functions, Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotonic functions, Infinite limits and limits at infinity.

15 Hours

Unit IV - Differentiation:

The derivative of a real function, Mean value theorems, The continuity of derivatives, L'Hospital's rule, Derivatives of higher order, Taylor's theorems, Differentiation of vector valued functions. 10 Hours

References:

1. Walter Rudin – Principles of Mathematical Analysis, McGraw Hill, 3rd Edition, 1976.
2. Robert. G. Bartle – The Elements of Real Analysis, Wiley International Edition, New York, 2nd Edition 1976.
3. T. M. Apostol – Mathematical Analysis, Addison /Wesley, Narosa, New Delhi, 2nd Edition, 1985.
4. Ajith Kumar and S. Kumaresan – A basic Course in Real Analysis, CRC Press, 2014.
5. R. R. Goldberg – Methods of Real Analysis, Oxford & I. B. H. Publishing Co., New Delhi, 2nd Edition, 1970.
6. N. L. Carothers – Real Analysis, Cambridge University Press, 2000.
7. Russel A. Gordon – Real Analysis - A first Course, Pearson, 2nd Edition, 2011.

MTS 404	Numerical Analysis	4 Credits (48 Hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Course Outcome: Students will have the knowledge and skills to explain the fundamental concepts of Numerical analysis, area of mathematics and computer science that creates, analyzes, and implements algorithms for obtaining numerical solutions to problems involving continuous variables. Such problems arise throughout the natural sciences, social sciences, engineering, medicine and business.

Course Specific Outcome: At the end of the course students will have the knowledge and skills

- Obtain the solutions of Transcendental and Polynomial Equations.
- Solve by Direct methods and Iteration methods for solving system of equations.
- Apply Hermite Interpolation
- Solve problems using interpolation.
- Solve Ordinary Differential Equations using Numerical methods.

Unit I- Transcendental and Polynomial Equations:

Introduction, The bisection method, Iteration methods based on first degree equation, Iteration methods based on second degree equation, Rate of convergence, Rate of convergence of Secant and Newton-Raphson method Iteration methods, First order method, Second order method, Higher order methods. Polynomial equations, Descartes' Rule of Signs, The Birge-Vieta method.

12 Hours

Unit II- System of Linear Algebraic Equations and Eigenvalue problems:

Introduction, Direct methods - Gauss elimination method, Gauss-Jordan method, Triangularization method, Cholesky method. Iteration methods - Jacobi iteration method, Gauss-Seidel iteration method, Convergence analysis, Eigenvalues and eigenvectors. The power method.

12 Hours

Unit III - Interpolation and Approximation:

Introduction, Lagrange and Newton interpolations, Linear and Higher order interpolation, Finite difference operators, Interpolating polynomials using finite differences, Hermite interpolation, Approximations.

12 Hours

Unit IV

Numerical Differentiation: Introduction, Methods based on Interpolation, Methods based on finite differences, Methods based on undetermined coefficients, Extrapolation methods.

Numerical Integration: Methods based on Interpolation, Newton-Cotes methods, Composite Integration Methods.

12 Hours

References:

1. M. K. Jain, S. R. K. Iyengar – R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International, 6th Edition 2012.
2. C. F. Gerald and P. O. Wheatly – Applied Numerical Analysis, Pearson Education, Inc., 1999.
3. A. Ralston and P. Rabinowitz – A First Course in Numerical Analysis, 2nd Edition, McGraw - Hill, New York, 1978.
4. K. Atkinson – Elementary Numerical Analysis, 2nd Edition, John Wiley and Sons, Inc., 1994.
5. P. Henrici – Elements of Numerical Analysis, John Wiley and Sons, Inc., New York, 1964.

MTS 405	Number Theory	4 Credits (48 Hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Course Outcome: Students will have the knowledge and skills to explain the fundamental concepts and development of Elementary Number Theory using axioms, definitions, examples, theorems and their proofs.

Course Specific Outcome: At the end of the course students will have the knowledge and skills to:

- Apply the method of solving Linear Diophantine equations, Primality testing and factorization.
- Find the Dirichlet product of arithmetical functions, Dirichlet inverses.
- Solve the Linear congruences, Polynomial congruences modulo p, Simultaneous linear congruences, Simultaneous non-linear congruences, and Solving congruences modulo prime powers.
- Apply the properties of Legendre's symbol , Gauss lemma,
- Understanding the Pythagorean triples and their classification, Fermat's Last Theorem.
- Solve Pell's equation by continued fractions.

Unit I

Divisibility and Primes (Ref. [1]): Recapitulation of Division algorithm, Euclid's algorithm, Least Common Multiples, Linear Diophantine equations. Prime numbers and Prime-power factorisations, Distribution of primes, Fermat and Mersenne primes, Primality testing and factorization.

Arithmetical Functions (Ref. [2, 1]): The Möbius function and its properties, Euler function, examples and properties, The Dirichlet product of arithmetical functions, Dirichlet inverses and the Möbius inversion formula. 14 Hours

Unit II - Congruences (Ref. [2, 1]):

Recapitulation of basic properties of congruences, Residue classes and complete residue systems, Linear congruences. Reduced residue systems and the Euler-Fermat theorem, Polynomial congruences modulo p and Langrange's theorem, Simultaneous linear congruences, Simultaneous non-linear congruences, An extension of Chinese Remainder Theorem, Solving congruences modulo prime powers. 10 Hours

Unit III - Quadratic Residues and Quadratic Reciprocity Law (Ref. [2]):

Quadratic residues, Legendre's symbol and its properties, Euler's criterion, Gauss lemma, The quadratic reciprocity law and its applications, The Jacobi symbol, Applications to Diophantine equations. 12 Hours

Unit IV - Sums of squares, Fermat's last theorem and Continued fractions (Ref. [1, 3]):

Sums of two squares, Sums of four squares, The Pythagoras theorem, Pythagorean triples and their classification, Fermat's Last Theorem (Case $n = 4$).

Recapitulation of Finite continued fractions, Infinite continued fractions, Representation of irrational numbers, Periodic continued fractions and quadratic irrationals, Solution of Pell's equation by continued fractions.

12 Hours

References

1. G. A. Jones and J. M. Jones – Elementary Number Theory, Springer UTM, 2007.
2. Tom M. Apostol – Introduction to Analytic Number Theory, Springer, 1989.
3. David M. Burton – Elementary Number Theory, McGraw-Hill, 7th Edition, 2010.
4. Niven, H.S. Zuckerman & H.L. Montgomery – Introduction to the Theory of Numbers, Wiley, 2000.
5. H. Davenport – The Higher Arithmetic, Cambridge University Press, 2008.

II SEMESTER

MTE 451	Discrete Mathematics and Applications	3 Credits (36 Hours)

Prerequisite: Basic Mathematics up to class XII/PU.

Course Outcome: Students will have the knowledge and skills to explain the concepts of Discrete Mathematics and to develop logical thinking and its application to computer science, to enhance one's skills in solving real life problems related to counting, discrete probability by applying various counting techniques, to illustrate applications of Boolean algebra and group theory in designing logic gates and coding theory.

Course Specific Outcome: At the end of the course students will have the knowledge and skills to:

- Apply basic number theory concepts like divisibility, modular arithmetic in solving congruences, changing the base of number system and their usage in cryptography.
- Solve many real life problems related to counting by the use of special tools like reoccurrence relations and generating functions.
- Design and simplify the logic gate networks by using lattices and Boolean algebra.
- Apply concept of groups in generating binary coding, decoding and also in error detection and error correction in the binary coding system.

Unit I - Number Theory and Cryptography:

Divisibility and Modular Arithmetic, Integer Representations and Algorithms, Primes and Greatest Common Divisors, Solving Congruences, Applications of Congruences, Cryptography.

8 Hours

Unit II - Counting Techniques:

The Basics of Counting, The Pigeonhole Principle, Permutations and Combinations, Binomial Coefficients and Identities, Generalized Permutations and Combinations, Applications of Recurrence Relations, Solving Linear Recurrence Relations, Recurrence Relations, Generating Functions, Principle of Inclusion–Exclusion, Applications of Inclusion–Exclusion.

12 Hours

Unit III - Order Relations and Structures:

Product Sets and Partitions, Relations, Properties of Relations, Equivalence Relations, Partially Ordered Sets, Extremal Elements of Partially Ordered Sets, Lattices, Finite Boolean Algebras, Functions on Boolean Algebras, Boolean Functions as Boolean Polynomials.

8 Hours

Unit IV - Groups and Coding Theory:

Binary Operations Revisited, Semigroups, Products and Quotients of Semigroups, Groups, Products and Quotients of Groups, Coding of Binary Information and Error Detection, Decoding and Error Correction. 8 Hours

References:

1. Kenneth H. Rosen – Discrete Mathematics and Its Applications, Tata Mc-Graw-Hill, 7th Edition, 2012.
2. Bernard Kolman, Robert C. Busby, Sharon Cutler Ross – Discrete Mathematical Structures, Prentice Hall, 3rd Edition, 1996.
3. Grimaldi R – Discrete and Combinatorial Mathematics -1, Pearson, Addison Wesley, 5th Edition, 2004.

MTH 452	Algebra – II	4 Credits (48 hours)

Course Outcome: Students will have the knowledge and skills to Apply the advanced topics viz., Unique factorization domains, Field theory and Galois Theory in Coding theory and Cryptography, and also in diverse situations in physics, chemistry and engineering etc.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to **e**xplain Demonstrate accurate and efficient use of the following advanced topics in various situations -

- Unique factorization domains,
- Euclidean domains,
- Fields(including finite fields), Algebraically closed fields,
- The fundamental theorem of algebra, Galois Theory.

Unit I - Factoring: (Ref [1])

Unique factorization domains, Euclidean domains, Content of polynomials, Primitive polynomials, Gauss lemma, Unique factorization in $R[x]$, where R is a U.F.D., Irreducibility test mod p, Eisenstein's criterion, Gauss primes. 18 Hours

Unit II - Fields: (Ref [1])

Algebraic and transcendental elements, The degree of a field extension, Finding the irreducible polynomial, Ruler and compass constructions, Isomorphism of field extensions, Adjoining roots, Splitting fields, Finite fields, Primitive elements, Algebraically closed fields, The fundamental theorem of algebra. 20 Hours

Unit III - Galois Theory: (Ref [2])

Automorphisms and Fields, Separable Extensions, Galois Theory, Illustrations of Galois Theory, Cyclotomic Extensions, Insolvability of the Quintic. 10 Hours

References:

1. Michael Artin – Algebra, Prentice Hall of India 2nd edition, 2013.
2. J. B. Fraleigh – A First Course in Abstract Algebra, Addison Wesley, 7th edition, 2003.
3. I. N. Herstein – Topics in Algebra, John Wiley & Sons, 2nd edition, 2006.
4. Joseph A. Gallian – Contemporary Abstract Algebra, Cengage Learning India, 8th edition, 2013.
5. Paul B. Garrett – Abstract Algebra, CRC press, 2007.
6. Thomas W. Hungerford – Algebra, Springer, 2004.
7. David S. Dummit and Richard M. Foote – Abstract Algebra, Wiley, 3rd edition 2004.
8. Serge Lang – Algebra, 3rd Edition, Springer, 2005.

MTH 453	Real Analysis – II	4 Credits (48 hours)

Course Outcome: Students will have the knowledge and skills to demonstrate a competence in formulating, analysing and solving problems in several core areas of higher level Real Analysis, Develop skills to work with Riemann Integrals, sequences and series of functions and their convergence, approximation theory like Weierstrass Theorem, differentiation of several variable functions.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to **e**xplain Demonstrate accurate and efficient use of the following advanced topics in various situations -

- The Riemann-Stieltjes, Integral, Rectifiable curves, Improper Integrals.
- Sequences and Series of Functions, Uniform convergence and continuity.
- Integration, differentiation, Equicontinuous families of functions.
- The Stone-Weierstrass theorem.
- Functions of several variables: Differentiation, The contraction principle, The inverse function theorem, The implicit function theorem.

Unit I

The Riemann-Stieltjes Integral (Ref: [1]): Definition and existence of integrals, Properties of integral, Integration and differentiation, Integration of vector-valued functions, Rectifiable curves. Improper Integrals (Ref: [3]) : Definition, Criteria for convergence, Interchanging derivatives and integrals. 20 Hours

Unit II- Sequences and Series of Functions: (Ref: [1])

Discussion of main problem, Uniform convergence, uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Equicontinuous families of functions, The Stone-Weierstrass theorem. 16 Hours

Unit III- Functions of several variables: (Ref: [1])

Differentiation, The contraction principle, The inverse function theorem, The implicit function theorem. 12 Hours

References:

1. Walter Rudin – Principles of Mathematical Analysis, McGraw Hill, 3rd Edition, 1976.
2. Robert. G. Bartle – The Elements of Real Analysis, Wiley International Edition, New York, 2nd Edition, 1976.
3. Serge Lang – Analysis I, Addison Wesley Publishing Company 1968.
4. T. M. Apostol – Mathematical Analysis, Addison /Wesley, Narosa, New Delhi, 2nd Edition, 1985.
5. R. R. Goldberg – Methods of Real Analysis, Oxford & I. B. H. Publishing Co., New Delhi, 2nd Edition, 1970.
6. Ajith Kumar and S. Kumaresan – A basic course in real analysis, CRC Press, 2014.

MTH 454	Topology	5 Credits (60 hours)

Course Outcome: To study topological spaces, continuous functions, connectedness, compactness, countability and separation axioms.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to **e**xplain Demonstrate accurate and efficient use of the following advanced topics in various situations -

- Elementary concepts, Open bases and open subbases, Weak topologies
- The function algebras $C(X, \mathrm{R})$ and $C(X, \mathrm{C})$
- Countability axioms and Separability axioms
- Urysohn's lemma, Tietze extension theorem, and the Urysohn imbedding theorem.
- Connected spaces, the components of a space, totally disconnected spaces, locally connected spaces.

Unit I - Topological Spaces:
The definition and some examples, Elementary concepts, Open bases and open subbases, Weak topologies, The function algebras $C(X, \mathbb{R})$ and $C(X, \mathbb{C})$.

15 Hours

Unit II - Compactness:
Compact Spaces, Product spaces, Tychonoff's theorem.

15 Hours

Unit III - Separation:
T_1 – Spaces and Hausdorff spaces, Completely regular spaces and Normal spaces, Urysohn's lemma and Tietze extension theorem, The Urysohn imbedding theorem. 15 Hours

Unit IV- Connectedness:
Connected spaces, The components of a space, Totally disconnected spaces, Locally connected spaces. 15 Hours

References:

1. G. F. Simmons – Introduction to Topology and Modern Analysis, Tata McGraw-Hill, 2004.
2. J. R. Munkres – Topology, Second Edition, Pearson Education, Inc, 2000.
3. S. Willard – General Topology, Addison Wesley, New York, 1968.
4. J. Dugundji – Topology, Allyn and Bacon, Boston, 1966.
5. J. L. Kelley – General Topology, Van Nostrand Reinhold Co., New York, 1955.

MTS 455	Linear Algebra – II	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTH 402 (Linear Algebra – I).

Course Outcome: Students will have the knowledge and skills to demonstrate a competence in formulating, analysing and solving problems in several core areas of higher level of Linear Algebra concepts- Bilinear, Symmetric forms, orthogonal basis, spectral theorems, theory of modules in solving integer system, Hilbert basis theorem, Structure theorem which have plenty of applications in Fourier analysis, Wavelet Theory, Mathematical Physics and Chemistry.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to **e**xplain Demonstrate accurate and efficient use of the following advanced topics in various situations –

- Bilinear Forms, Hermitian forms
- Orthogonal Projection
- The spectral theorem,
- Skew symmetric forms, Modules, Free modules
- Diagonalizing Integer Matrices
- Noetherian Rings
- The structure theorem for abelian groups
- Application to linear operators.

Unit I - Bilinear Forms:

Bilinear forms, Symmetric forms, Hermitian forms, Orthogonality, Orthogonal Projection, Euclidean and Hermitian spaces, The spectral theorem, Skew symmetric forms, Summary of results in matrix notation. 24 Hours

Unit II - Linear Algebra in a Ring:

Modules, Free modules, Diagonalizing Integer Matrices, Submodule of free modules, Generators and Relations, Noetherian Rings, The structure theorem for abelian groups, Application to linear operators. 24 Hours

References:

1. Michael Artin – Algebra, Prentice Hall of India, 2nd Edition, 2013.
2. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence- Linear Algebra, Prentice Hall of India, 4th Edition, 2014.
3. K. Hoffmann and R. Kunz – Linear Algebra, Prentice Hall of India, 2nd Edition, 2013.
4. S. Lang – Linear Algebra, Addison Wesley, London, 1970.
5. Larry Smith – Linear Algebra, Springer Verlag, 3rd Edition, 1998.
6. Gilbert Strang – Linear Algebra and its Applications, Cengage Learning, 4th Edition, 2006.
7. S. Kumaresan – Linear Algebra- A Geometric Approach, PHI, 2003.

MTS 456	Ordinary Differential Equations	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabi prescribed for the courses MTH 402 (Linear Algebra – I) and MTH 403 (Real Analysis – I).

Course Outcome: Students will have the knowledge and skills of solving ordinary differential equations, boundary value problems, finding power series solutions of ordinary differential equations.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to **e**xplain Demonstrate accurate and efficient use of the following advanced topics in various situations –

- Notion of Linear dependence and the Wronskian
- The Basic theory for linear equations
- Solving differential equations using Power Series method
- The Legendre polynomials, Bessel's functions
- Solving Systems of first order equations
- Existence and uniqueness theorem.
- The fundamental matrix, Non-homogeneous linear systems, Linear systems with periodic coefficients.

Unit I - Linear Differential Equations of Higher Order:
Linear dependence and the Wronskian, Basic theory for linear equations, Method of variation of parameters, Reduction of n^{th} order linear homogeneous equation, Homogeneous and non-homogeneous equations with constant coefficients.

10 Hours

Unit II - Solutions in Power Series:
Second order linear equations with ordinary points, Legendre equation and Legendre polynomials, Second order equations with regular singular points, Bessel equation.

20 Hours

Unit III - Systems of Linear Differential Equations:
Systems of first order equations, Existence and uniqueness theorem. The fundamental matrix, Non-homogeneous linear systems, Linear systems with periodic coefficients.

10 Hours

Unit IV - Existence and Uniqueness of solutions :
Equations of the form $x' = f(t,x)$, Method of successive approximation, Lipschitz condition, Picards theorem, Non uniqueness of solutions, Continuation of solutions.

8 Hours

References:

1. S. G. Deo and V. Raghavendra – Ordinary Differential Equations and Stability Theory, Tata McGraw Hill, 1980.
2. A. Coddington – An Introduction to Ordinary Differential Equations, Prentice Hall of India, 2013.
3. A. Coddington and N. Levinson – Theory of Ordinary Differential Equations, Krieger, 1984.

4. M. W. Hirsh and S. Smale – Differential Equations, Dynamical Systems and Linear Algebra, Academic Press, New York, 1974.
5. V. I. Arnold – Ordinary Differential Equations, MIT Press, Cambridge, 1981.
6. Shepley L. Ross – Differential Equations, Wiley, 2004.

III SEMESTER

MTE 501	Differential Equations and Applications	3 Credits (36 Hours)

Prerequisite: Basic Mathematics up to class XII/PU.

Course Outcome: Students will have the knowledge and skills to apply the theory of differential equations in formulating many fundamental laws of physics and chemistry, set up second order differential equations in different models to describe damped/undamped vibrations and forced vibrations and derive properties of Special Functions of Mathematical Physics like Bessel functions, Legendre polynomials, etc.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to

- Illustrate the applications of theory of differential equations in economics and biology to model the behaviour of complex systems.
- Create and analyze mathematical models using first and second order differential equations to solve application problems such as mixture problems, population modeling harmonic oscillator and LCR circuits.
- Describe solutions of differential equations by the use of Laplace transforms and study the properties of special functions of mathematical physics through series solutions.

Unit I:

Recapitulation of methods of solutions of first order differential equations, Applications of First Order Ordinary Differential Equations - Simple problems of dynamics – falling bodies and other motion problems, Simple problems of Chemical reactions and mixing, Simple problems of growth and decay. 10 Hours

Unit II:

Applications of Second Order Ordinary Differential Equations - Undamped simple harmonic motion, damped vibrations, Forced vibrations, Problems on simple electric circuits – Laplace transforms. 10 Hours

Unit III:

Power series solutions of Second Order Linear Differential Equations, their mathematical properties. Special Functions of Mathematical Physics - Bessel functions, Legendre polynomials, Chebyshev polynomials, Hermite polynomials and Laguerre polynomials. 16 Hours

References:

1. G. F. Simmons – Differential Equations with Applications and Historical Notes, Tata McGraw-Hill, New Delhi, 1991.
2. D. Rainville and P. Bedient – Elementary course on Ordinary Differential Equations, Macmillan, New York, 1972.
3. R. Courant and D. Hilbert – Methods of Mathematical Physics, Vol. I, Tata McGraw Hill, New Delhi, 1975.

MTH 502	Complex Analysis – I	5 Credits (60 Hours)

Course Outcome: Students will have the knowledge and skills to apply the theory of complex analysis course in - engineering and allied sciences. This course is a foundation for next course in Complex analysis.

Course Specific Outcome:
At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- the need for a Complex Number System
- The stereographic projection,
- Analytic functions, Sequences,
- Series, Uniform convergence, Power series.
- The exponential and trigonometric functions,
- Cauchy's theorem, Cauchy's Integral Formula,
- Removable singularities, Taylor's theorem, Zeros and poles,
- The maximum principle.

Unit I - Complex numbers and Complex Functions:
Recapitulation of the algebra of complex numbers - Arithmetic operations, Square roots, Conjugation, Absolute value, Inequalities.
The geometric representation of complex numbers - Geometric addition and multiplication, The binomial equation, Analytic geometry, The spherical representation.
Introduction to the concept of analytic function - Limits and continuity, Analytic functions, Polynomials, Rational functions.
Elementary theory of power series - Sequences, Series, Uniform convergence, Power series, Abel's limit theorem.
The exponential and trigonometric functions - The exponential, The trigonometric functions, The periodicity, The logarithm. 20 Hours

Unit II - Analytic Functions as Mappings, Complex Integration :
Elementary Point set Topology - All topological properties to be reviewed, with an emphasis on Connectedness, and compactness.
Conformality - Arcs and closed curves, Analytic functions in regions, Conformal mapping, Length and area.
Linear transformation - The linear group. The cross ratio, Symmetry.
Fundamental theorems - Line integrals, Rectifiable arcs, Line integrals as function of arcs, Cauchy's theorem for a rectangle, Cauchy's theorem for a disk.
Cauchy's Integral Formula - The index of a point with respect to a closed curve, The integral formula, Higher derivatives. 20 Hours

Unit III - Local Properties of Analytical Functions:
Removable singularities, Taylor's theorem, Zeros and poles, The local mapping, The maximum principle.
The General Form of Cauchy's Theorem: Chains and cycles, Simple connectivity, Homology, The general statement of Cauchy's theorem: Cauchy's theorem (Statement only). Locally exact differentials, Multiply connected regions. 20 Hours

References:

1. Lars V. Ahlfors – Complex Analysis, McGraw Hill, 3rd Edition, 1979.
2. B. R. Ash – Complex Variables, Dover Publications, 2nd Edition, 2007.
3. R. V. Churchill, J. W. Brown and R. F. Verlag – Complex Variables and Applications, Mc Graw Hill, 8th Edition, 2009.
4. J. B. Conway – Functions of one Variable, Narosa, New Delhi, 1996.
5. S. Ponnuswamy and H. Silverman – Complex Variables with Applications, Birkhäuser, 2006.

MTH 503	Measure and Integration	5 Credits (60 Hours)

Course Outcome: Students will have the knowledge and skills to apply the Measure Theory. The concepts are very much applicable in probability theory in Statistics.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts –

- Lebesgue outer measure, Lebesgue measure, and Lebesgue measurable functions.
- Fatou’s lemma, Monotone convergence theorem, and Lebesgue Dom- inated convergence theorem.
- Characterize Riemann integrable functions on [a, b].
- Vitali Covering lemma, Lebesgue theorem.
- Functions of bounded variation, Absolutely continuous function, and their importance in the study of differentiation of an integral.
- The extension theorem of Caratheodary.
- Product measure and Fubini theorem.

Unit I

Algebras of sets - Borel sets. Outer measure, Measurable sets and Lebesgue measure. Example of a non-measurable set. Measurable functions.

15 Hours

Unit II

The Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, The integral of a nonnegative function, The general Lebesgue integral.

15 Hours

Unit III

Differentiation and Integration, Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity.

15 Hours

Unit IV

Measure and outer measure, The extension theorem of Caratheodary, The product measures, The Fubini theorem.

15 Hours

References:

1. H. L. Royden – Real Analysis, Prentice - Hall, 3^{rd} Edition, 2003.
2. G. D. Barra – Introduction to Measure Theory, Van Nostrand Reinhold Company Ltd., 1974.
3. Walter Rudin – Real and Complex Analysis, Tata McGraw Hill Publishing Company, 3^{rd} Edition, 1987.
4. P. R. Halmos – Measure Theory, Springer Verlag, 1974.
5. F. Hewitt and K. Stromberg – Real and Abstract Analysis, Springer Verlag, 1965.
6. Inder K. Rana – An Introduction to Measure and Integration, Narosa Publishing House, 2^{nd} Edition, 1997.

7.

MTH 504	Multivariate Calculus and Geometry	4 Credits (48 Hours)

Course Outcome: Students will have the knowledge and skills to work with level sets, tangent spaces, maxima and minima of several variable functions, to develop theory of integrals – surface integrals, volume integrals etc and Greens theorem, Stoke's theorem , theory of geometry surfaces, Curvatures, Geodesic etc.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- Find Level sets and tangent spaces
- Apply Lagrange multipliers method
- Find Maxima and minima on open sets
- Evaluate Line Integrals
- Apply Green's theorem
- Evaluate Surface area, Surface integrals
- Apply Stoke's theorem, the divergence theorem.
- Understand the geometry of surfaces in R^3, Gaussian Curvature, Geodesic.

Unit I
Introduction to differentiable functions, Level sets and tangent spaces, Lagrange multipliers, Maxima and minima on open sets.

10 Hours

Unit II
Curves in R^3 , Line Integrals, The Frenet-Serret equations, Geometry of curves in R^3.

12 Hours

Unit III
Double integration- Green's theorem. Parametrised surfaces in R^3, Surface area, Surface integrals, Stoke's theorem, Triple integrals, The divergence theorem.

12 Hours

Unit IV
The geometry of surfaces in R^3, Gaussian Curvature, Geodesic. 14 Hours

<u>References</u>:

1. Sean Dineen – Multivariate Calculus and Geometry, Springer Undergraduate Mathematics Series, 3rd Edition, 2014.
2. Andrew Pressly – Elementary Differential Geometry, Springer Undergraduate Mathematics Series, 2nd Edition, 2010.
3. Walter Rudin – Principles of Mathematical Analysis, McGraw Hill, New York, 3rd Edition, 1976.
4. J. A. Thorpe – Elementary Topics in Differential Geometry, Undergraduate Texts in Mathematics, Springer Verlag, 1994.
5. W. Klingenberg – A course in Differential Geometry, Springer Verlag, 1983.

MTS 505	Advanced Numerical Analysis	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 404 (Numerical Analysis).

Course Outcome: Students will have the knowledge and skills of Numerical Integration, Numerical solutions of Ordinary Differential Equations, Solving systems of Linear Differential Equations.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- To use different quadrature rules for computing the approximate value of definite integrals
- To use different numerical techniques to solve ordinary differential equations with initial and boundary conditions.
- To use different methods to find numerical solution of second order partial differential equations.

Unit I - Numerical Integration:

Recapitulation of the methods based on interpolation, Methods based on undetermined coefficients. Romberg integration, Gauss-Legendre integration methods, Gauss-Chebyshev integration methods, Gauss-Laguerre integration methods, Gauss-Hermite integration methods. Double integration, Trapezoidal rule, Simpson's rule.

15 Hours

Unit II - Ordinary Differential Equations:

Introduction, Numerical methods, Euler method, Backward Euler method, Mid-point method, Single step methods, Taylor series method, Runge-Kutta methods, Multistep methods, Determination of a_j and b_j, Predictor-corrector methods, Boundary value problems, Difference methods, Boundary value problems for $y'' = f(x, y)$. Trapezoidal, Dahlquist and Numerov methods.

15 Hours

Unit III - Numerical Solution of Second Order Partial Differential Equations:

Introduction, Difference methods, Parabolic equations in one space dimension, Schmidt formula, Du Fort-Frankel scheme, Crank-Nicolson and Crandall schemes, Solution of hyperbolic equation in one dimension by explicit schemes, The CFL condition, Elliptic equations, Dirichlet problem, Neumann problem, Mixed problem.

18 Hours

References:

1. M. K. Jain, S. R. K. Iyengar, P. K.Jain – Numerical Methods for Scientific and Engineering Computation, New Age International, 6th Edition, 2012.
2. C. F. Gerald and P. O. Wheatly – Applied Numerical Analysis, Pearson Education, Inc., 1999.
3. M. K. Jain – Numerical Solution of Differential Equations, New Age International (P) Ltd., New Delhi, 2nd Edition , 1984.
4. A. R. Mitchell – Computational Methods in Partial Differential Equations, John Wiley and Sons, Inc., 1969.

MTS 506	Commutative Algebra	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabus prescribed for the courses MTH 401 (Algebra – I) and MTH 452 (Algebra – II).

Course Outcome: The course is a comprehensive introduction to commutative rings and modules. It is meant to give students a foundation for further studies in algebra and algebraic geometry.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- Zero divisors, Nilpotent elements, Units, Prime ideals and maximal ideals,
- Nilradical and Jacobson radical in a ring
- Operations on ideals, Extensions and contraction of ideals.
- Nakayama's lemma
- local properties, Extended and contracted ideals in rings of fractions
- First and second uniqueness theorems, the going-up and going-down theorems
- Primary decomposition in Noetherian rings.

Unit I - Rings and ideals:

Zero divisors, Nilpotent elements, Units, Prime ideals and maximal ideals, Nilradical and Jacobson radical, Operations on ideals, Extensions and contraction of ideals.

18 Hours

Unit II - Modules:

Recapitulation of Operations on submodules, Isomorphism theorems. Direct sum and product, Finitely generated modules, Nakayama's lemma, Exact sequences (omit tensor products and related results).

08 Hours

Unit III- Rings and modules of fractions:

Local properties, Extended and contracted ideals in rings of fractions.

08 Hours

Unit IV- Primary decomposition, Integral dependence and chain conditions:

First and second uniqueness theorems, Integral dependence, The going-up theorem, Integrally closed integral domains, The going-down theorem, Noetherian rings and modules, Primary decomposition in Noetherian rings.

14 Hours

References:

1. M. F. Atiyah and I. G. Macdonald – Introduction to Commutative Algebra, Lavant Books (Indian Edition), 2007.
2. N. Bourbaki – Commutative Algebra, American Mathematical Society, 1972.
3. N. S. Gopalkrishnan – Commutative Algebra, University Press, 2nd edition, 2015.
4. G. Northcott – Lesson on Rings, Modules and Multiplicities, Cambridge University Press, 2008.
5. O. Zariski and P. Samuel – Commutative Algebra Vol I, II, Graduate Texts in Mathematics, Springer Verlag, 1976.

MTS 507	Graph Theory	4 Credits (48 Hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Course Outcome: Graph Theory is an integral part of Discrete Mathematics and has applications in diversified areas such as Electrical Engineering, Computer science, Linguistics. Students will have the knowledge and skills to apply the concepts of Trees, Eulerian Graphs, Matching, Vertex colorings, Planarity.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to **e**xplain Demonstrate accurate and efficient use of the following topics in various situations -

- The problem of Ramsey
- Extremal graphs, Operations on graphs,
- Menger's theorem
- Traversability and Planarity,
- Eulerian graphs, Hamiltonian graphs.
- Coloring
- Matrices associated with graphs.

Unit I - <u>Graphs:</u>

Varieties of graphs, Walks and connectedness, Degrees, The problem of Ramsey, Extremal graphs, Intersection graphs, Operations on graphs.

8 Hours

Unit II - <u>Blocks, Trees and Connectivity:</u>

Cut points, Bridges, Blocks, Block graphs and Cut point graphs, Characterization of trees, Centers and centroids, Block-Cutpoint trees, Independent cycles and cocycles. Connectivity and line-connectivity, Graphical variations of Menger's theorem

18 Hours

Unit III - <u>Traversability and Planarity:</u>

Eulerian graphs, Hamiltonian graphs. Plane and planar graphs Outerplanar graphs,

10 Hours

Unit IV

<u>Colorability</u>: The chromatic number, The Five Color Theorem, The chromatic polynomial.

<u>Matrices</u>: The adjacency matrix, The incidence matrix and The cycle matrix, Matrix-Tree Theorem.

12 Hours

<u>References:</u>

1. F. Harary – Graph Theory, Addison-Wesley Series in Mathematics, 1969.
2. Narsingh Deo – Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India, 1987.
3. Bela Bollabas – Modern Graph theory, Springer, 1998.
4. R.Balakrishnan and K.Ranganathan – A textbook of Graph Theory, Springer-Verlag, 2000.
5. Douglass B. West – Introduction to Graph Theory, Prentice Hall of India, New Delhi, 1996.
6. O. Ore – Theory of Graphs, American Mathematical Society, Providence, Rhode Island, 1967.

MTS 508	Lattice Theory	4 Credits (48 hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate Level.

Course Outcome: Students will have the knowledge and skills to apply the concepts of Partially Ordered Sets, Lattices in General , Complete Lattices, Distributive and Modular Lattices, and Complemented Modular Lattices and Boolean Algebras. The concepts of lattice theory are applied in various field within mathematics and allied subjects like Quantum mechanics in Physics and concept lattices in computer science.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to **e**xplain Demonstrate accurate and efficient use of the following topics in various situations -

- Partially ordered sets, Axiom of choice, Zorn's lemma and Hausdorff's maximal chain principle,
- Duality principle, Ideals, Atomic lattices,
- Complemented, Complete lattices, Distributive, Modular lattices and their Characterizations
- The isomorphism theorem, The prime ideal theorem, Boolean algebras.

Unit I - Partially ordered sets:

Partially ordered sets (or Posets), Diagrams, Lower and upper bounds, Order homomorphism and order isomorphism, Special subsets of a poset. Axiom of choice (Statement only). Zorn's lemma and Hausdorff's maximal chain principle, and proof of the equivalence of these two statements. Length of a poset, The minimum and maximum conditions, Duality principle for posets (Topics selected from Chapter I of [1]). 10 Hours

Unit II - Lattices in general:

A lattice as a poset and as an algebra, Diagrams of lattices, Duality principle for lattices, Semilattices, Sublattices, Ideals and prime ideals of lattices, Ideal generated by a nonempty subset of a lattice and its description, The ideal lattice and the augmented ideal lattice of a lattice, Bound elements, atoms and dual atoms in a lattice, Atomic lattices, complemented, relatively complemented and sectionally complemented lattices, Homomorphisms, congruence relations and quotient lattices of lattices, The homomorphism theorem. (Topics selected from Chapter II of [1] and Chapter I of [2]). 12 Hours

Unit III - Complete lattices:

Complete lattices, fixed point property. Compact elements and compactly generated lattices (Topics selected from Chapter III of [1] and Chapter 2 of [3]). 6 Hours

Unit IV- Distributive and modular lattices:

Distributive, Modular lattices, Characterizations of modular and distributive lattices in terms of sublattices, The isomorphism theorem of modular lattices, The prime ideal theorem for distributive lattices (Topics selected from Chapters IV and VIII of [1] and Chapter II of [2]).

12 Hours

Unit V - Complemented modular lattices and Boolean algebras:

Complemented modular lattices and bounded relatively complemented lattices. Distributivity of a uniquely complemented relatively complemented lattice, Boolean algebras, De Morgan formulae, Boolean algebras and Boolean rings, Distributive lattices and rings of sets, Boolean algebras and fields of sets (Topics selected from Chapter V, VI and VIII of [1]). 8 Hours

References:

1. G. Szasz – Introduction to Lattice Theory, Academic Press, N.Y., 1963.
2. G. Gratzer – General Lattice Theory, Birkhauser Verlag, Basel, 1978.
3. P. Crawley and R.P, Dilworth – Algebraic Theory of Lattices, Prentice - Hall Inc., N. J., 1973.
4. G. Birkhoff – Lattice Theory, American Mathematical Society Colloquium Publications, Volume 25, 1995.
5. L. A. Skornjakov – Elements of Lattice Theory, Hindustan Publishing Corporation, 1977.

MTS 509	Fluid Mechanics	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 456 (Ordinary Differential Equations).

Course Outcome: This course is intended to provide a treatment of topics in fluid mechanics to a standard where the student will be able to apply the techniques used in deriving a range of important results and in research problems. It provides the student with knowledge of the fundamentals of fluid mechanics and an appreciation of their application to real world problems.

Course Specific Outcome:

At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- Fundamentals of Fluid Mechanics,
- Develop understanding about hydrostatic law, principle of buoyancy and stability of a floating body and application of mass, momentum and energy equation in fluid flow.
- Imbibe basic laws and equations
- Fluid flow measurement
- The losses in a flow system, flow through pipes, boundary layer flow and flow past immersed bodies.

Unit-I

Motion of inviscid fluids:- Recapitulation of equation of motion and standard results - Vortex motion-Helmholtz vorticity equation - Permanence of vorticity and circulation - Kelvin's minimum energy theorem – Impulsive motion - Dimensional analysis - Nondimensional numbers.

6 Hours

Unit-II

Two dimensional flows of inviscid fluids:- Meaning of two-dimensional flow - Stream function – Complex potential - Line sources and sinks - Line doublets and vortices - Images - Milne-Thomson circle theorem and applications - Blasius theorem and applications.

12 Hours

Unit-III

Motion of Viscous fluids:- Stress tensor – Navier-Stokes equation - Energy equation - Simple exact solutions of Navier-Stokes equation: (i) Plane Poiseuille and Hagen- Poiseuille flows (ii) Generalized plane Couette flow (iii) Steady flow between two rotating concentric circular cylinders (iv) Stokes's first and second problems (vi) Slow and steady flow past a rigid sphere and cylinder. Diffusion of vorticity - Energy dissipation due to viscosity. Boundary layer concept - Derivation of Prandtl boundary layer equations - Blasius solution – Karman's integral equation.

16 Hours

Unit-IV

Gas Dynamics:-Compressible fluid flows - Standard forms of equations of state - Speed of sound in gas - Equations of motion of non-viscous and viscous compressible flows. Subsonic, sonic and supersonic flows - Isentropic flows - Gas dynamical equations.

7 Hours

Unit-V

Turbulent Flow:- Introduction – Transition from laminar to turbulent flow - Homogeneous turbulence – Isotropic turbulence — Spatial, time and ensemble averages – Basic properties of averages – Reynolds averaging procedure – Derivation of turbulent equations using Reynolds averaging procedure with gradient-diffusion i.e., K- model for closure.

7 Hours

References:

1. F. Chorlton–Text book of Fluid Dynamics, Van Nostrand, 1967.
2. L. M. Milne –Thomson – Theoretical Hydrodynamics, Macmillan, 4^{th} Edition, 1960.
3. S. W. Yuan – Foundations of Fluid Mechanics, Prentice Hall, 1976.
4. Z. U. A.Warsi – Fluid Dynamics, CRC Press, 2^{nd} Edition, 1999.
5. B.K. Shivamoggi–Theoretical Fluid Dynamics, John Wiley and Sons, 1998.
6. Stephen B. Pope, Turbulent Flows, CambridgeUniversity Press, 2000.
7. C.S. Yih – Fluid Mechanics, McGraw-Hill, 1969.
8. E.L. Cussier – Difussion Mass Fluid Systems, 2^{nd} Edition, Cambridge University Press, 2006.

MTS 510	Theory of Partitions	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 405 (Number Theory).

Course Outcome: This course motivates students towards research in the theory of partitions in the spirit of Ramanujan, whose contribution in the field is remarkable. Students will have the knowledge and skills to extensive use of generating functions and Ferrer's graph to derive properties of partition function, apply concepts of q-series to derive famous results theory of partitions like Jacobi's triple product identity, Ramanujan's 1ψ1 - summation formula, the Rogers - Ramanujan Identities and exposed to Ramanujan's work on number theory and some open problems in the field to carry the research in the field.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- generating functions
- q-series
- Ramanujan's 1ψ1 - summation formula,
- the Rogers - Ramanujan Identities.
- Restricted partitions, Gauss polynomials and Gaussian coefficients and their applications.

Unit I

Partitions - partitions of numbers, the generating function of p(n), other generating functions, two theorems of Euler, Jacobi's triple product identity and its applications.

14 Hours

Unit II

1ψ1 - summation formula and its applications, combinatorial proofs of Euler's identity, Euler's pentagonal number theorem, Franklin's combinatorial proof.

12 Hours

Unit III

Congruence properties of partition function, the Rogers - Ramanujan Identities.

10 Hours

Unit IV

Elementary series - product identities, Euler's, Gauss', Heine's, Jacobi's identities. Restricted Partitions – Gaussian, Frobinius partitions.

12 Hours

References:

1. G. H. Hardy and E. M. Wright – An Introduction to Theory of Numbers, Oxford University Press, 5th Edition, 1979.
2. I. Niven, H. S. Zuckerman and H. L. Montgomery – An Introduction to the Theory of Numbers, New York, John Wiley and Sons, Inc., 5th Edition , 2004.
3. Bruce C. Berndt – Ramanujan's Note Books Volumes-1 to 5.
4. G. E. Andrews – The Theory of Partitions, Addison Wesley, 1976.
5. A. K. Agarwal, Padmavathamma, M. V. Subbarao – Partition Theory, Atma Ram & Sons, Chandigarh, 2005.

IV SEMESTER

MTH 552	Complex Analysis – II	4 Credits (48 Hours)

Course Outcome: Students will have the knowledge and skills to use complex analysis techniques to get asymptotics, to be rational and get real, solve analytic combinatorics viz,. the calculus of residues, Poisson's formula, Schwarz's theorem, the reflection principle, the Fourier development, the Weierstrass $\wp$ function. Complex analysis has several applications to the study of Banach algebras in Functional analysis, Holomorphic functional calculus, and Control theory.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- Evaluation of definite integrals.
- Harmonic Functions, Poisson's formula, Schwarz's theorem, The reflection principle. Power series expansions - Weierstrass's theorem, The Taylor series
- The Laurent series. Partial fractions, Infinite products
- The Gamma function, Jensen's formula, Product development of Riemann Zeta function.
- Elliptic Functions.

Unit I

The Calculus of Residues: The Residue theorem, The argument principle, Evaluation of definite integrals.

Harmonic Functions: Definition and basic properties, The mean value property, Poisson's formula, Schwarz's theorem, The reflection principle.

14 Hours

Unit II - Series and Product Developments:

Power series expansions - Weierstrass's theorem, The Taylor series, The Laurent series.

10 Hours

Unit III - Partial Fractions and Faclorization:

Partial fractions, Infinite products, Canonical products, The Gamma function, Jensen's formula, Product development of Riemann Zeta function.

12 Hours

Unit IV

Elliptic Functions:

Simply periodic functions - Representation by exponentials, The Fourier development, Function of finite order.

Doubly Periodic Functions: The period module, Unimodular transformation, General properties of elliptic functions. The Weierstrass $\wp$ function.

12 Hours

References:

1. Lars V. Ahlfors – Complex Analysis, McGraw Hill, 3rd Edition, 1979.
2. B. R. Ash – Complex Variables, Dover Publications, 2nd Edition, 2007.
3. R. V. Churchill, J. W. Brown and R. F. Verlag – Complex Variables and Applications, Mc Graw Hill, 8th Edition, 2009.
4. J. B. Conway – Functions of one Variable, Narosa, New Delhi, 1996.
5. S. Ponnuswamy and H. Silverman – Complex Variables with Applications, Birkhäuser, 2006.

MTH 553	Functional Analysis	4 Credits (48 Hours)

Course Outcome: Students will have the knowledge and skills to explain and apply the concepts: Baire's theorem, Banach spaces, Continuous linear transformations, the Hahn Banach theorem, the open mapping theorem, Uniform boundedness principle, Hilbert spaces, and Normal and unitary operators.These concepts are useful in Fourier analysis, wavelet and curvelet theories and also in Quantum mechanics.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts-

- To develop basic understanding of the theory of Banach spaces, continuous linear transformations, Hahn Banach Theorem etc.
- To study the basics of Hilbert spaces, orthonormal sets, The conjugate of a Hilbert space etc.
- To understand the theory of adjoint operators, Normal operators, Finite dimensional spectral theorem etc.

Unit I

Review of metric spaces: Convergence, Completeness and Baire's theorem.

Banach spaces: Definition and some examples, Continuous linear transformations, The Hahn Banach theorem, The natural embedding of N in N**, The open mapping theorem, Uniform boundedness principle. 24 Hours

Unit II - Hilbert spaces:

Definition and examples, Orthogonal complements, Orthonormal sets, The conjugate of a Hilbert space, The adjoint operator, Self-adjoint operators, Normal and unitary operators, Projections, Finite dimensional spectral theorem.

24 Hours

References:

1. G. F. Simmons – Introduction to Topology and Modern Analysis, McGraw Hill, 2004.
2. A. E. Taylor, David Lay –Introduction to Functional Analysis, John Wiley and Sons, 1980.
3. Ward Cheney – Analysis for Applied Mathematics, Graduate Texts in Mathematics, Springer, 2001.
4. Walter Rudin – Real and Complex Analysis, McGraw Hill, 3rd Edition, 1986.
5. M. Thamban Nair – Functional Analysis A First Course, Prentice-Hall, 2002.

MTS 554	Partial Differential Equations	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 456 (Ordinary Differential Equations).

Course Outcome: Students will have the knowledge and skills of solving partial differential equations with different techniques.

Course Outcome/Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the techniques to-

- Solve differential equation of the form $dx/P = dy/Q = dz/R$, Pfaffian differential equations
- Find Orthogonal trajectories of a system of curves on a surface.
- Solve linear equations and Nonlinear equations of order one.
- Study the Dirichlet problem for a rectangle, Neumann problems
- Solve Laplace equation in Cylindrical and Spherical coordinates.
- Solve diffusion equation in Cylindrical and spherical coordinates.
- Solve Initial value problem - D'Alembert's solution, Vibrating string
- Solve Boundary and initial value problems for two dimensional wave equation.

Unit I

Ordinary differential equations in more than two variables: Recapitulation of Methods of solution of $dx/P = dy/Q = dz/R$, Pfaffian differential forms and Pfaffian differential equations and solutions. Orthogonal trajectories of a system of curves on a surface.

First order partial differential equations: Origin of first order partial differential equations, The Cauchy problem for first order equations, Linear equations of first order, Integral surfaces passing through a given curve, Surfaces orthogonal to a given system of surfaces, Nonlinear equations of first order, Cauchy's method of characteristics, Charpit's method, Special types of first order equations. 24 Hours

Unit II

Higher Order Partial Differential Equations: Linear partial differential equations with constant coefficients, Classification of second order PDE, Canonical forms, Adjoint operators, Riemann's method.

Elliptic Differential Equations: Dirichlet problem for a rectangle, Neumann problem for a rectangle, interior and exterior Dirichlet problem for a circle, Interior Neumann problem for a circle. Solution of Laplace equation in Cylindrical and Spherical coordinates.

Parabolic Differential Equations: Occurrence of the diffusion equation, Elementary solutions of the diffusion equation, Dirac Delta function, Separation of variables, Solution of diffusion equation in Cylindrical and spherical coordinates.

Hyperbolic Differential Equations: Solution of one dimensional equation by canonical reduction, Initial value problem - D'Alembert's solution, Vibrating string - variable separation method, Forced vibrations, Boundary and initial value problems for two dimensional wave equation, Uniqueness of the solution for the wave equation, Duhamel's principle.

24 Hours

References:

1. Ian Sneddon – Elements of Partial Differential Equations, Mc-Graw Hill, International student edition, 1957.
2. K. Sankara Rao – Introduction to Partial Differential Equations, Prentice-Hall of India, 1995.
3. F. John – Partial Differential Equations, Springer Verlag, New York, 1971.
4. P. Garabedian – Partial Differential Equations, Wiley, New York, 1964.
5. R. Chester – Techniques in Partial Differential Equations, McGraw Hill, New York, 1971.

MTS 555	Advanced Topology	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTH 454 (Topology).

Course Outcome: Students will have the knowledge and skills of advances in point-set topology and Algebraic Topology.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts-

- Minimal uncountable well ordered set S_Ω
- Order topology
- The box and product topologies
- Compact sets in ordered sets having the least upper bound property
- Countability properties of spaces R_l, R_l^2, I_o^2, S_Ω and $\overline{S_\Omega}$.
- Separation properties of spaces R_K, S_Ω and $S_\Omega \times \overline{S_\Omega}$, Imbeddings of manifolds
- The Nagata-Smirnov Metrization Theorem.
- Paracompactness.
- The Homotopy and the fundamental group.

Unit I - Preliminaries:

Order relations and dictionary order relations, Well ordering theorem, The minimal uncountable well ordered set S_Ω and its basic properties (Ref: relevant topics from §3 and §10 of [1]). The order topology and the ordered square I_o^2, the least upper bound property of I_o^2. Box and product topologies on arbitrary products of spaces and continuity of a function from a space into these products (Ref: relevant topics from §19 of [1]). Compact sets in ordered sets having the least upper bound property (Ref: relevant topics selected from §27 of [1]), Equivalence of compactness, limit point compactness and sequential compactness in metrizable spaces (Ref: §28 of [1]).

12 Hours

Unit II - Countability and separation axioms:

The countability axioms and their properties, study of countability properties of spaces R_l, R_l^2, I_o^2, S_Ω and $\overline{S_\Omega}$. The separation axioms and their properties, separation properties of spaces R_K, S_Ω and $S_\Omega \times \overline{S_\Omega}$. Urysohn lemma(Statement only), Imbedding theorem and Urysohn Metrization theorem, Partitions of unity (finite case), Imbeddings of manifolds (Ref: §30, §31, §32, §34 and §36 of [1]).

12 Hours

Unit III - Metrization theorems and paracompactness:

Local finiteness. The Nagata-Smirnov Metrization Theorem. Paracompactness. (Ref: Chapter 6 of [1] up to Theorem 41.5).

12 Hours

Unit IV - The fundamental group and covering spaces:

Homotopy of paths, The fundamental group, Covering spaces, The fundamental group of the circle. (Ref: Chapter 9 of [1], up to Theorem 54.5).

12 Hours

References:

1. J. R. Munkres – Topology Second Edition, Pearson Education, Inc, 2000.
2. G. F. Simmons – Introduction to Topology and Modern Analysis, Tata McGraw-Hill, 2004.
3. S. Willard – General Topology, Addison Wesley, New York, 1968.
4. J. Dugundji –Topology, Allyn and Bacon, Boston, 1966.
5. J. L. Kelley – General Topology, Van Nostrand Reinhold Co., New York, 1955.
6. E. H. Spanier – Algebraic Topology, McGraw-Hill, 1966.

MTS 556	Advanced Discrete Mathematics	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTH 401 (Algebra – I).

Course Outcome: Students will have the knowledge and skills to develop techniques for constructing mathematical proofs, illustrated by discrete mathematics examples, to design and simplify the logic gate networks by using lattices and Boolean algebra and Karnaugh Maps, and highlight some important applications of graph theory in the development of algorithms in rooting and designing computer network finding optimal solutions to some construction problems.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts-

- Advanced Counting Principles to solve problems on combinatorics.
- The Polya's counting principle and Polya's inventory problems to solve the problems on coloring.
- Design and simplify the logic gate networks by using lattices and Boolean algebra and Karnaugh Maps.
- Solving problems on extremal graph theory and develop DFS, BFS, and Shortest Path Algorithms.

Unit – I

Basic Counting Principles: Number of one-one functions, Permutations, Combinations, Number of onto functions. Partitions and Stirling Numbers of Second kind.

Advanced Counting: Pigeon-hole Principle, Inclusion-Exclusion Principle, Putting Balls into boxes, Round Table Configurations, Counting using Lattice Paths, Catalan Numbers. Recurrence Relations, Generating Functions, Using generating functions to prove results related to certain binomial coefficients.

18 Hours

Unit-II - Applications of Group Theory (Ref [1]): Recapitulation of Group Action, Orbit Stabilizer Theorem and its applications to Polya's Counting Principle (Polya's Theorem (Special Case) and Polya's Theorem (General Case)) and Polya's Inventory Problems.

10 Hours

Unit-III - Boolean Algebras and Switching Functions (Ref. [4], [3]):

Introduction, Boolean Algebras, Boolean Functions, Switching Mechanisms, Minimization of Boolean Functions, Applications to Digital Computer Design. Switching Functions: Disjunctive and Conjunctive Normal Forms, Gating Networks, Minimal sums of products, Karnaugh Maps and further Applications.

10 Hours

Unit-IV- Graph Theory (Ref [6], [7]):

Introduction, Matching and Factorization. Extremal Graph Theory - Turans Theorem. DFS, BFS, Shortest Path Algorithms. 10 Hours

References:

1. D. I. A. Cohen – Basic Techniques of Combinatorial Theory, John Wiley and Sons, New York, 1978.
2. G. E. Martin – Counting: The Art of Enumerative Combinatorics, UTM, Springer, 2001.

3. Ralph P. Grimaldi – Descrete Combinatorial Mathematics, Pearson, 5th Edition, 2006.
4. Mott J. L. , Kandel A. and Baker T. P. – Discrete Mathematics for Computer Scientists and Mathematicians, Second Edition, Prentice Hall India, 1986.
5. Kenneth H. Rosen – Discrete Mathematics and its Applications, McGraw Hill 7th Edition, 2012.
6. F. Bukley and Frank Harary – Distance in Graphs, Addition Wisley Publishing Comany, 1990.
7. G. Chartrand, L. Lesniak and P. Zhang – Graphs and Digraphs, CRC Press, 5th Edition, 2011.

MTS 557	Algebraic Number Theory	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabi prescribed for the courses MTH 452 (Algebra – II) and MTS 506 (Commutative Algebra).

Course Outcome: Students will have the knowledge and skills to apply the concepts of the course in
Advanced level of Mathematics related to algebraic number theory including Dedekind's zeta function.

Course Specific Outcome:
At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- Algebraic and transcendental numbers,
- Algebraic Number Fields,
- Algebraic Integers, Integral Bases, Norms and Traces,
- Factorizations,
- The Ramanujan-Nagell Theorem,
- Dedekind domains, Ramification index and degree of a prime ideal, The splitting of rational primes in algebraic number fields,
- Class group and class number.

Unit I - Algebraic Numbers:
Recapitulation of Field Extensions and properties, Definition and Examples of algebraic and transcendental numbers, Liouville's Theorem, Algebraic Number Fields, Conjugates and Discriminants, Algebraic Integers, Integral Bases, Norms and Traces, Rings of Integers, Quadratic Fields and Cyclotomic Fields.

12 Hours

Unit II - Factorization into Irreducibles:
Trivial Factorizations, Factorization into Irreducibles, Examples of Non-Unique Factorization into Irreducibles, Prime Factorization, Euclidean Domains, Euclidean Quadratic Fields, Consequences of Unique Factorization, The Ramanujan-Nagell Theorem.

12 Hours

Unit III - Factorization of Ideals:
Dedekind domains, Fractional ideals, Invertible ideals, Prime factorization of ideals, Congruences, Norm of an ideal, Ideals in different number fields, Ramification index and degree of a prime ideal, The splitting of rational primes in algebraic number fields, Slplitting of primes in quadratic fields.

16 Hours

Unit IV - Class group and class number:
Definition of the Class group and class number, Minkowski's theorem, Finiteness of the class-group, Class number computations.

8 Hours

References:

1. I. N. Stewart and David Tall – Algebraic Number Theory and Fermat's Last Theorem, A. K. Peters Ltd., 2002.
2. Jody Esmonde and M. Ramamurthy – Problems in Algebraic Number Theory, Springer Verlag 2nd Edition 2004.
3. Pierre Samuel – Algebraic Theory of Numbers, Dover Publications, 2008.
4. Karlheinz Spindler – Abstract Algebra with Applications, Vol. II, Rings and Fields, Marcel Dekkar, Inc, 1994.
5. Saban Alaca and Kenneth S. Williams – Introductory Algebraic Number Theory, Cambridge University Press, 2004.

MTS 558	Calculus of Variations and Integral Equations	4 Credits (48 Hours)

Prerequisite: Knowledge of syllabus prescribed for the course MTS 456 (Ordinary Differential Equations).

Course Outcome: Students will have the knowledge and skills to apply the concepts of the course in – solving difficult popular problems arising in Physics, Chemistry, Engineering and technology, Statistical Analysis, and also in Economics.

Course Specific Outcome:

At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- Solving the problem of brachistochrone, problem of geodesics, isoperimetric problem, Variation and its properties, functions and functionals,
- Solving Variational problems with the fixed boundaries, and Moving boundary problems
- One-sided variations, conditions for one sided variations.
- Variational problems involving conditional extremum, constraints involving several variables and their derivatives, Isoperimetric problems.
- The Conversion of Volterra Equation to ODE, IVP and BVP to Integral Equation.
- The Fredholm's first, second and third theorem, Integral Equations with symmetric kernel, Eigen function expansion, Hilbert-Schmidt theorem.

Unit I : Variational problems with the fixed boundaries

Introduction, problem of brachistochrone, problem of geodesics, isoperimetric problem, Variation and its properties, functions and functionals, Comparison between the notion of extrema of a function and a functional. Variational problems with the fixed boundaries, Euler's equation, the fundamental lemma of the calculus of variations, examples, Functionals in the form of integrals, special cases containing only some of the variables, examples, Functionals involving more than one dependent variables and their first derivatives, the system of Euler's equations, Functionals depending on the higher derivatives of the dependent variables, Euler-Poisson equation, examples, Functionals containing several independent variables, Ostrogradsky equation, examples.

12 Hours

Unit II: Variational problems with moving boundaries, Sufficiency conditions:

Moving boundary problems with more than one dependent variables, transversality condition in a more general case, examples, Extremals with corners, refraction of extremals, examples, One-sided variations, conditions for one sided variations. Field of extremals, central field of extremals, Jacobi's condition, The Weierstrass function, a weak extremum, a strong extremum, The Legendre condition, examples, Transforming the Euler equations to the canonical form, Variational problems involving conditional extremum, examples, constraints involving several variables and their derivatives, Isoperimetric problems, examples.

12 Hours

Unit III – Integral Equations:

Introduction, Definitions and basic examples, Classification, Conversion of Volterra Equation to ODE, Conversion of IVP and BVP to Integral Equation.

Fredholm’s Integral equations - Decomposition, direct computation, Successive approximation, Successive substitution methods for Fredholm Integral Equations.

10 Hours

Unit IV

Voltera Integral equations: A domain decomposition, series solution, successive approximation, successive substitution method for Volterra Integral Equations, Volterra Integral Equation of first kind, Integral Equations with separable Kernel.

Fredholm’s theory - Hilbert-Schmidt theorem: Fredholm's first, second and third theorem, Integral Equations with symmetric kernel, Eigenfunction expansion, Hilbert-Schmidt theorem.

Fredholm and Volterra Integro-Differential equation: Fredholm and Volterra Integro-Differential equation, Singular and nonlinear Integral Equation.

14 Hours

References:

1. Curant, R. and D. Hilbert – Methods of Mathematical Physics, Vol I. Interscience Press, 1953.
2. Elsgolc, L.E. –Calculus of Variations, Pergamon Press Ltd., 1962.
3. Weinstock, Robert – Calculus of Variations with Applications to Physics and Engineering, Dover, 1974.
4. Porter, D. and Stirling, D. S. G. – Integral Equations, A practical treatment from spectral theory and applications, Cambridge University Press, 1990.
5. Ram. P. Kanwal – Linear Integral Equations Theory and Practise, Academic Press 1971.
6. A. M. Wazwaz –A first course in integral equations, World Scientific Press, 1997.
7. Cordumeanu C. – Integral Equations and Applications, Cambridge University Press, 1991.

MTS 559	Mathematical Statistics	4 Credits (48 Hours)

Prerequisite: Knowledge of Mathematics at Under-Graduate level.

Course Outcome: Students will have the knowledge and skills to develop the concept of Probability, Conditional Probability and Moments to study the different statistical models, describe the use of probability distributions and functions of random variables in the study of sampling distributions and their properties, and illustrate testing of hypotheses statistical inference to summarize the main features of a data set and study the behaviors of the collected data.

Course Specific Outcome: At the end of the course Students will have the knowledge and skills to understand, explain in depth and apply in various situations the concepts -

- A probability generating function, a moment generating function, and a cumulant generating function and cumulants.
- Apply central limit theorem, and explain the concepts of random sampling, statistical inference and sampling distribution, and use basic sampling distributions.
- Describe the main methods of estimation and the main properties of estimators, and apply them.
- Use different testing hypothesis like MP test, Likelihood ratio tests, t- test, Chi-square test, Wilcoxon sign rank test, and Run test etc.

Unit I - Probability, Conditional Probability and Moments:
Sample space, class of events; Classical and Axiomatic definitions of Probability, their consequences. Conditional Probability, Independence, Bayes' theorem and applications. Random Variables, Distributions Functions, Probability Mass functions, Probability Density functions. Expectations, Moment generating function, Probability generating function, Chebyshev's and Jensen's inequalities and applications.

12 Hours

Unit II - Distributions:
Standard discrete distribution and their properties – Bernoulli, Binomial, Geometric, Negative Binomial, Poisson distributions. Standard continuous distribution and their properties – Uniform, Exponential, Normal, Beta, Gamma distributions. Functions of random variables – transformation technique and applications, Sampling distributions – t, Chi-square, F and their properties.

14 Hours

Unit III - Random Sequences, Statistical Inference and Testing Hypothesis :
Sequences of random variables - Convergence in distribution and in probability, Chebyshev's, Weak law of large numbers. Central limit theorem and applications. Point estimation - sufficiency, unbiasedness, method of moments, maximum likelihood estimation.
Testing of hypotheses – Basic concepts, Neyman-Person lemma, MP test. Likelihood ratio tests, t-test, Chi-square test and their applications. Nonparametric tests and their applications – Sign, Wilcoxon sign rank test, Run test.

22 Hours

References:

1. Rohatgi V. K. – An introduction to probability theory and mathematical statistics, Wiley Eastern ltd, 1985.
2. Bhat B. R. – Modern Probability Theory, an introductory text, Wiley eastern Ltd, 1981.

3. Robert B Ash –Probability and Mathematical Statistics, Academic Press, Inc. NY, 1972.
4. Hogg R.V. and Craig A. T. – Introduction to Mathematical Statistics, McMillan and Co., 6^{th} Edition, 2004.
5. E. L. Lehmann and J. P. Romano –Testing Statistical Hypothesis, Springer, 3^{rd} Edition, 2005.
6. Freund, J.F. – Mathematical Statistics, Prentice Hall India, 8th Edition, 2012.

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<u>Semester wise distribution of credits for M.Sc. Mathematics Programme</u>

Sem	Hard Core			Soft Core			Open Elective			Project	Total
	No. of Courses	Credits	Total Credits	No. of Courses	Credits	Total Credits	No. of Courses	Credits	Total Credits		
I	3	5	15	2	4	8					23
II	2	4	13	2	4	8	1	3	3		24
	1	5									
III	1	4	14	2	4	8	1	3	3		25
	2	5									
IV	2	4	8	2	4	8				4	20
Total			50			32			6*	4	86+6*

*Not included for CGPA

Total of Hard Core Credits is 50+4 = 54 and Total Soft Core Credits is 32.

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